This volume represents the proceedings of the 3rd International Symposium on Numerical Models in Geomechanics (NUMOG III) held on 8–11 May 1989, Niagara Falls, Canada.

Technical Advisory Committee

W. D. L. Finn	Canada
H. B. Poorooshasb	Canada
P. K. Banerjee	USA
Z. Bažant	USA
C. S. Desai	USA
J. Ghaboussi	USA
S. Nemat-Nasser	USA
J. R. Booker	Australia
Z. Mroz	Poland
G. Gudehus	FR Germany
K. Ishihara	Japan

Numerical Models in Geomechanics

NUMOG III

Edited by

S. Pietruszczak

McMaster University, Hamilton, Ontario, Canada

and

G. N. Pande

University College of Swansea, Swansea, UK

ELSEVIER APPLIED SCIENCE
LONDON and NEW YORK

NUMERICAL MODELLING OF A CIRCULAR FOUNDATION OVER VIBROFLOTED SAND

GIOVANNI CANETTA
Ce.A.S. Center for Structural Analysis s.r.l.
10 Viale Giustiniano, 20129 MILAN, ITALY

ROBERTO NOVA
Department of Structural Engineering
Milan University of Technology (Politecnico)
32, Piazza Leonardo da Vinci, 20133 MILAN, ITALY

ABSTRACT

Vibrofloted sand is considered as a composite material made of densified sand and gravel columns. Starting from the constitutive equations of sand and gravel, a procedure is presented to derive the constitutive law of an equivalent homogeneous medium. The settlements of an uniformly loaded circular foundation on vibrofloted sand are next analyzed. The influence of different design parameters such as width and depth of vibrofloted zone is investigated. It is shown that a key parameter that controls calculated settlements is the K_0 value after soil treatment.

INTRODUCTION

The mechanical characteristics of loose sandy deposits are often improved by vibroflotation. With that technique, the natural soil is densified by vibration and strengthened by columns which are formed by filling the vibroflot holes with gravel or other coarse material.

Although vibroflotation has been used for many years, design methods are still based on semi-empirical rules [1],[2]. Only a rough estimation of settlements is possible in this way and no rules exist to cope with eccentric or inclined loading or disuniformities in the subsoil. Moreover no indication is given on the effect of design parameters such as width and depth of the vibrofloted zone.

Recently, Schweiger and Pande [3] proposed a method of analysis of clayey soils reinforced by stone columns in which the reinforced soil is treated as a composite material. The idea is that of considering the improved subsoil as a homogeneous medium of average characteristics. The overall stiffness of the homogeneized medium, $\underline{D}^h$, is in fact obtained as a linear combination of the soil stiffness $\underline{D}^s$ and the stiffness of the column material, $\underline{D}^c$:

$$\underline{D}^h = \rho\, \underline{D}^c + (1-\rho)\, \underline{D}^s \tag{1}$$

where ρ is the replacement ratio, i.e. the ratio between the cross section area of the column and the plan area of natural soil per column.

D^c and D^s depend on the stress and strain history. It is assumed that natural stresses are the same in the column and the surrounding clay, while vertical and shear stresses may be different. Total strains are supposed to be identical for the two materials.

In this paper a similar path of reasoning will be followed, in the attempt of modelling the behaviour of a foundation over vibrofloted sand.

It will be assumed, moreover, that strains and stresses in the sand and in the gravelly column are such that equilibrium and compatibility at the interface between the two soil types are fulfilled.

HOMOGENEIZATION PROCEDURE

The consitutive law of an element of soil can be written in incremental terms as

$$\dot{\underline{\sigma}}^{c,s} = \underline{D}^{c,s}\dot{\underline{\epsilon}}^{c,s} \tag{2}$$

Soil and column material occupy adjacent regions of space. Within each region equation (2) holds with either the apex c or the apex s, and stresses and strains vary with the position. Across the interface between soil and column, however, the stress and stress rate components should be equal to satisfy equilibrium requirements. Similarly the strain and strain rate components in the direction of the interface should be equal to fulfil compatibility. Stresses and strains in soil and column are therefore not independent from each other.

Equality in stress and strain components is rigorously valid only at the interfaces. In order to define an equivalent homogeneous medium, however, we shall enforce this condition all over the vibrofloted zone, as if columns were smeared over it. In each point we shall then define an overall stress vector Σ and an overall strain vector E which are related to the stresses and strains within soil and column material. They can be partitioned in two subvectors:

$$\underline{\Sigma} = \begin{Bmatrix} \Sigma_E \\ \hline \Sigma_C \end{Bmatrix} \qquad \underline{E} = \begin{Bmatrix} E_E \\ \hline E_C \end{Bmatrix} \tag{3}$$

where the indices E and C stand for equilibrium and compatibility, respectively.

The subvector Σ_E groups all the stress components that should be equal by virtue of equilibrium at interface, that is

$$\underline{\Sigma}_E = \begin{Bmatrix} \Sigma_x \\ \Sigma_y \\ T_{xy} \\ T_{xz} \\ T_{yz} \end{Bmatrix} = \underline{\sigma}_E^c = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^c = \underline{\sigma}_E^s = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^s \tag{4}$$

where the z axis is parallel to the direction of the column axis. E_C groups all the strain components that should be equal by virtue of compatibility. Thus:

$$\underline{E}_C = \{E_z\} = \underline{\epsilon}_C^c = \{\epsilon_z\}^c = \underline{\epsilon}_C^s = \{\epsilon_z\}^s \tag{5}$$

The other stress and strain components in soil and column material are independent. To define an equivalent homogeneized medium various ways can be followed. We shall impose that the overall second order work in the homogeneized medium is equal to the sum of the second order works in soil and column:

$$\int_V \tilde{\dot{\underline{\Sigma}}}\dot{\underline{E}}\,dV = \int_{V_c} \tilde{\dot{\underline{\sigma}}}^c \dot{\underline{\epsilon}}^c dV_c + \int_{V_s} \tilde{\dot{\underline{\sigma}}}^s \dot{\underline{\epsilon}}^s dV_s \tag{6}$$

where a superposed tilde means transpose. Since the volume considered is of infinitesimal dimensions, because of the smearing of the columns, the strain and stress rate vectors are constant over it. It follows that

$$\tilde{\dot{\underline{\Sigma}}}\dot{\underline{E}} = \rho \tilde{\dot{\underline{\sigma}}}^c \dot{\underline{\epsilon}}^c + (1-\rho)\tilde{\dot{\underline{\sigma}}}^s \dot{\underline{\epsilon}}^s \tag{7}$$

From equations (4) and (5) we may finally derive

$$\begin{cases} \dot{\underline{\Sigma}}_C = \rho \dot{\underline{\sigma}}_C^c + (1-\rho)\dot{\underline{\sigma}}_C^s \\ \dot{\underline{E}}_E = \rho \dot{\underline{\epsilon}}_E^c + (1-\rho)\dot{\underline{\epsilon}}_E^s \end{cases} \tag{8}$$

By imposing the equilibrium conditions (4) and the compatibility condition (5), taking account of equations (8), it is possible to express the strain rates in soil and column material as a linear combination of the overall strain rates $\dot{\underline{E}}$, [4].

$$\dot{\underline{\epsilon}}^{c,s} = \underline{T}_\epsilon^{c,s}\dot{\underline{E}} \tag{9}$$

The matrices $T_\epsilon^{c,s}$ depend on the stiffness matrices $D^{c,s}$ and the replacement ratio ρ. Taking account of equations (9) in equation (7) we can finally derive the expression of the stiffness of the homogeneized medium as a function of the individual stiffnesses of its components:

$$\underline{D}^h = \rho \tilde{\underline{T}}_\epsilon^c \underline{D}^c \underline{T}_\epsilon^c + (1-\rho)\tilde{\underline{T}}_\epsilon^s \underline{D}^s \underline{T}_\epsilon^s \tag{10}$$

This equation can be compared with equation (1).

The solution procedure for a boundary value problem is then as follows. Given the state of stress in soil and column, evaluate the initial stiffness matrices D^c, D^s. Next calculate $T_\epsilon^{c,s}$ and D^h via equation (10). Solve then the b.v.p. as for a homogeneous medium and obtain the strain increments $\dot{E}$ from the compatibility equations. Evaluate further the strain increments in soil and column by means of equation (9) and the corresponding stress increments via the appropriate constitutive law. Update stiffness matrices and start loop again.

If the stiffness matrices depend on the direction of vector $\dot{\underline{\sigma}}^{c,s}$, a procedure similar to the initial stress method may be followed for each component until the desired level of convergence is achieved.

CONSTITUTIVE MODEL FOR SAND AND GRAVEL

The constitutive model that will be employed is elastic-plastic strainhardening. The yield function is given by:

$$f = 3/2(\xi-1)J_{2\eta} - \xi J_{3\eta} + 3(\xi-3)\ln(p'/p_c) = 0 \qquad (11)$$

where ξ is a constitutive parameter related to the angle of friction at constant volume, p' is the mean effective pressure, p_c is the maximum hydrostatic preconsolidation pressure, $J_{2\eta}$ and $J_{3\eta}$ are the second and third invariants of the tensor η_{ij}, defined as

$$\eta_{ij} = s_{ij}/p' \qquad (12)$$

$$J_{2\eta} = \eta_{ij}\eta_{ij} \qquad (13)$$

$$J_{3\eta} = \eta_{ij}\eta_{jk}\eta_{ki} \qquad (14)$$

The tensor s_{ij} is the stress deviator.

The function f is bounded by a limiting surface given by the Matsuoka-Nakai (1974) failure condition which can be written as

$$3/2(\xi-1)J_{2\eta} - \xi J_{3\eta} - 3(\xi-3) = 0 \qquad (15)$$

Equations (11) and (15) concide when $p'=p_c/e$. The constitutive law employed is therefore a sort of cap-model.

For the sake of convenience, normality rule is assumed to be valid. In order to avoid unrealistic dilations, however, on the limiting surface the plastic strain rate vector is assumed to be normal to the yield surface given by equation (11).

Hardening is assumed to depend on both volumetric and deviatoric plastic strains, as suggested by Nova (1977) and Wilde (1977):

$$p_c = p_0 \exp[1/\chi(\epsilon^p_{rs}\delta_{rs} - D\sqrt{J_{2e}})] \qquad (16)$$

$$J_{2e} = e^p_{rs}e^p_{rs} \qquad (17)$$

where p_0 is a reference pressure, χ is the plastic logarithmic compliance, D is a dilatancy parameter while ϵ^p_{rs} and e^p_{rs} are the plastic strain and deviator strain tensors, respectively. At variance, with Cam Clay and similar models, with that assumption it is possible to model the dilation of a sample while the state of stress is still in the hardening region.

Plastic strain rates can be derived via the Prager's consistency rule.

For a virgin soil, failure will occur when the hardening modulus is equal to zero. It can be shown that this occurs when

$$\xi J_{3\eta} - (\xi-1)J_{2\eta} + \xi - 3 + \frac{D}{\sqrt{J_{2e}}}[(\xi-1)\eta_{ij}e^p_{ij} - \xi\eta_{il}\eta_{lj}e^p_{ij}] = 0 \qquad (18)$$

It can be shown that D is related to the dilation at failure. From equation (18) an apparent friction angle ϕ'_f can be calculated.

Note that equation (18) does not coincide with equation (15). In this model, the Matsuoka-Nakai condition is therefore not intended as a failure condition but as a limit surface, which is associated to a negative hardening modulus. Therefore if the stress-path touches the limit surface, softening occurs.

Fig. 1 shows the aspect of yield surface, limit surface and failure condition (H=0) in the so called triaxial plane. Yield surface is also shown in the deviatoric plane.

To get total strains, elastic strains are added to plastic ones. They are characterized by a tangent bulk modulus linearly increasing with isotropic pressure and by a constant shear modulus G.

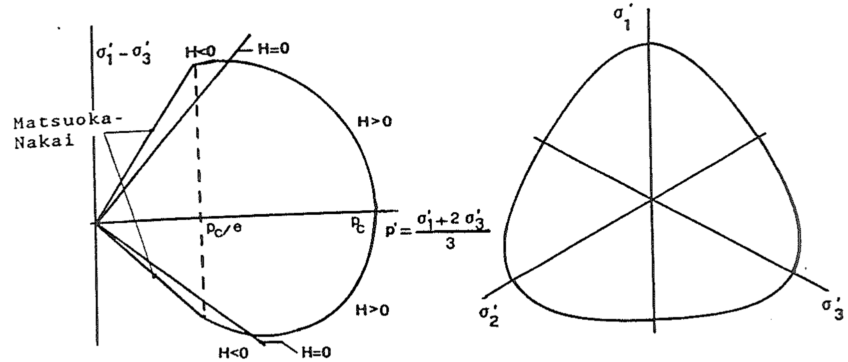

Figure 1. Yield, limit and failure surface in the triaxial and deviatoric planes.

SETTLEMENTS OF A UNIFORMLY LOADED CIRCULAR FOUNDATION

A very stiff circular foundation, resting over a deposit of loose sand partially improved by vibroflotation, is loaded by a vertical force. The size of the foundation and of the vibrofloted zone, the replacement ratio and the value of the uniform lateral surcharge are shown in Fig. 2. The material parameters characterizing soil types are given in Table 1.

TABLE 1

type of soil	D	ϕ'_{cv}	ϕ'_f	χ	K/p'	ν_0	p_0(kPa)	$\gamma(kN/m^3)$
gravel	.130	32°	40°	.001	800	.25	5	20
natural sand	0	32°	32°	.005	200	.25	5	14
v.floted sand	.071	32°	35°	.003	400	.25	5	17

It is in fact assumed that vibroflotation densifies the sand beneath the foundation so that its stiffness and strength parameters increase. Moreover vibroflotation changes the initial state of stress in the treated sand by inducing a field of horizontal selfstresses. It is assumed that the value of K_0 in sand after treatment is 3 times the value of natural soil. Indeed even larger values have been measured [5].

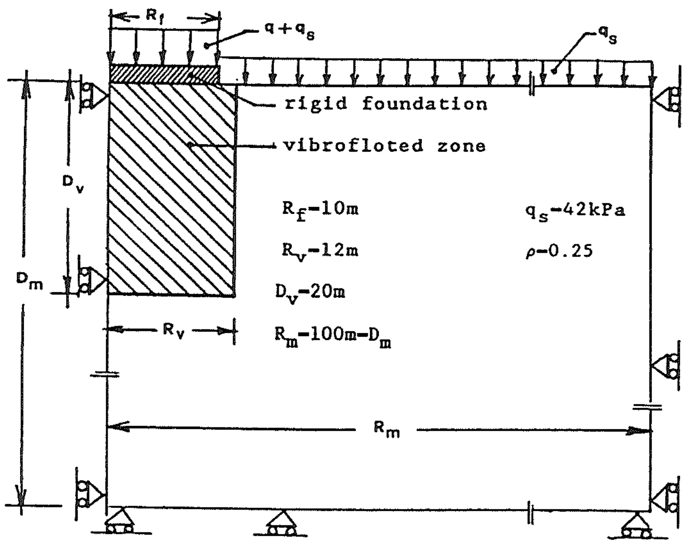

Figure 2. Circular, stiff foundation on vibrofloted sand.

First, foundation settlements up to failure were computed. Figure 3 shows a comparison of the results obtained for reinforced and unreinforced soil. In practice, for such a large mat, only the initial part of the curve

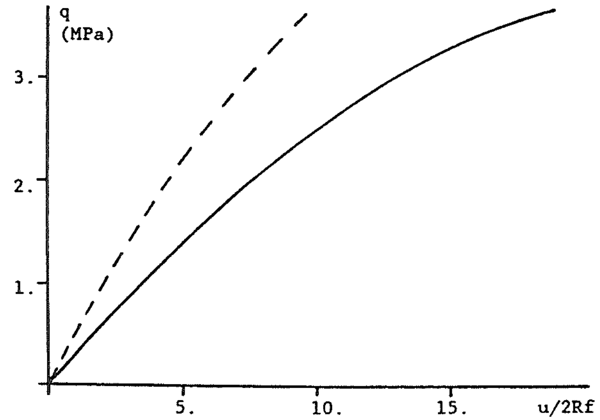

Figure 3. Settlement computation for unreinforced (solid line) and reinforced (dotted line) soil.

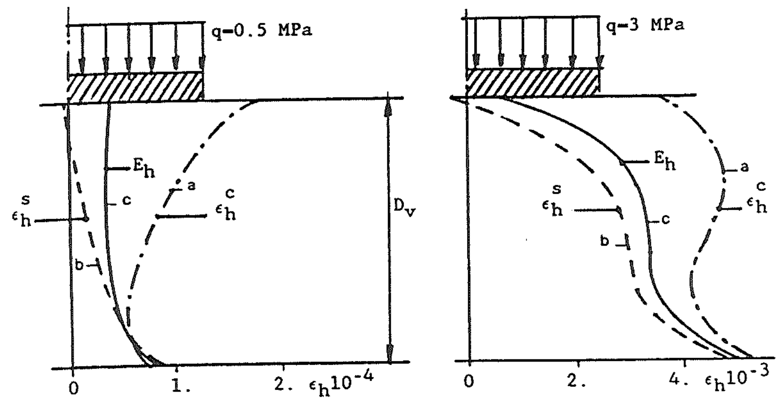

Figure 4. Horizontal strains in a) gravel and b) sand and homogeneized horizontal strain c), beneath foundation center at various loading levels.

is relevant since settlements become rapidly untolerable. Up to .75 MPa settlements are roughly proportional to the applied loading. The settlement reduction factor is of the order of 2 and tends to diminish with increasing working level since the beneficial effects of vibroflotation tend to be erased by the yielding of soil. The computed concentration factor in columns is only 1.75. Fig. 4 shows the computed horizontal strains in the column and in the improved soil beneath the foundation center, as a variation with respect to homogeneized strain. The main features of column bulging become evident only for very high loading levels.

Next a parametric analysis was performed to investigate the influence of different design parameters. A major role is played by the initial state of stress. If it is assumed that vibroflotation densifies the soil but does not generate a change in the stress field, computed settlements at low working level are as much as 40% larger than in the former case. This difference drops to 12% when the loading reaches .75 MPa.

If the vibrofloted zone is widened, foundation settlements become smaller. The cost of vibroflotation increases faster than the reduction in settlement. The former is in fact related to the area of the treated zone, while settlements diminish linearly with the increase of the radius. In the case considered, however, the deepening of the treated zone is even less efficient. If column length is increased from 20 m to 35 m, the gain in settlement reduction is 5% only.

The influence of the column geometry was also investigated. Settlements are only 8% smaller if replacement ratio is doubled. A negligible difference exist if, by taking constant the volume of the treated zone and of the gravel employed, ρ is not constant. If the soil beneath the central part of the foundation has a replacement ratio which is twice the ratio of the peripheral part of the foundation, the settlement of the center is 2% less than in the case of uniform column distribution. The settlement is instead 2% more if ρ at the periphery is twice ρ in the center.

Finally, if instead of making an analysis with the homogeneization method, settlements were computed as if all the soil beneath the foundation had the characteristics of the vibrofloted zone, the center of the foundation would settle 20% less than in the former case. The latter type of analysis is therefore not conservative.

CONCLUSIONS

Starting from the constitutive equations of soil and column material, it is possible to calculate the stiffness matrix of an equivalent homogeneized material and solve a boundary value problem. The equivalence is obtained by enforcing the fulfilment of equilibrium and compatibility at interface and by imposing that the second order work in the equivalent material is equal to the sum of the work done in soil and column material.

By employing an elastic-plastic strainhardening constitutive model for sand and gravel, a parametric analysis was performed to investigate the influence of design parameters on computed settlements. Such analyses cost only a little more than analyses over a homogeneous soil, since no column discretization is necessary.

It is shown that the key parameter in controlling settlements is a correct determination of the K_0 value after soil treatment.

ACKNOWLEDGEMENTS

R.Nova is indebted to MPI for financial support.

REFERENCES

1. Schulze G., Consolidation des sols par vibration profonde et par compactage dinamique, C.R. Symp. 'La consolidation des sols et des roches in situ dans le Genie Civil', Stresa, 1978, pp. 83-115.

2. Aboshi H.,Ichimoto E., Enoki M. and Harada K., The 'composer'. A method to improve characteristics of soft clays by inclusion of large diameter sand colums', Int. Conf. Soil Reinforcement, Paris, 1979, 1, pp. 271-277.

3. Schweiger H.F. and Pande G.N., Numerical analysis of stone column supported foundations, Computers and Geotechnics, 1986, pp. 347-372.

4. Canetta G. and Nova R., A numerical method for the analysis of ground improved by columnar inclusions, Computers and Geotechnics, 1989, to appear.

5. Saito A., Characteristics of penetration resistance of a reclaimed sandy deposit and their change through vibratory compaction, Soils and Foundations, 1977, 17, 4, pp. 32-43.