# A METHOD FOR ANALYSIS AND DESIGN FOR FLEXIBLE RETAINING STRUCTURES

# UNE MÉTHODE POUR L'ANALYSE ET LE DIMENSIONNEMENT DES STRUCTURES DE SOUTÈNEMENT FLEXIBLE

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A new method for stress analysis of sheetpiles and prediction of displacements is presented. Soil is modelled as a bed of springs, whose constitutive law is elastic plastic strainhardening. All the constitutive parameters are linked to usual geotechnical parameters. The comparisons performed show a good agreement between experimental data of reduced scale models and numerical predictions.

Cette communication présente une nouvelle méthode pour l'analyse de l'état de contrainte et la prévision des déplacements des palplanches. Le sol est schématisé à l'aide d'un lit des ressorts élastoplastiques écrouissables. Tous les paramètres constitutifs sont liés aux paramètres géotechniques usuels. On montre, à l'aide de plusieurs comparaisons, qu'il existe un bon accord entre les résultats expérimentaux obtenus sur modèles réduits et les visions des calculs.

#### INTRODUCTION

The analysis and design of anchored sheetpiles walls is tackled either by means of semiempirical rules or via a numerical analysis. The latter method is certainly more rational and allows to take into account soil nonhomogenheities without difficulties, but it is not free from shortcomings. In the simplest approach, soil is idealized as a bed of elastic springs. The difficulty, however, is connected to the choice of the correct value for the stiffness of the springs. Since this method oversimplifies the problem, the choice of the stiffness value depends not only on the soil but also on the geometrical and mechanical characteristics of the problem at hand, such as the depth and stiffness of the wall, so that the spring stiffness is different for springs at the back or in front of the sheetpile. The best choice for predicting bending moments in the structure may not be the best for determining displacements. On the other hand, the finite element method allows a more thoroughful investigation of the state of stress and strain in the soil and in the retaining structure, but a finite element analysis is costly and time consuming, especially if a realistic constitutive law for soil is employed. It can be used for verification of an already designed structure, but design optimization is virtually impossible for cost reasons.

The method that will be presented here tries to overcome such difficulties combining the simplicity of the spring model with a more adequate idealization of soil behaviour, which is considered as elastic plastic strainhardening. It will be shown that it is possible to obtain reasonable predictions for soil pressures, bending moments, sheetpile displacements and to follow the variation of the tensile force in the anchors with the depth of the excavation. The accuracy of predictions is good not only in normal working conditions, but even the depth of the excavation that produces failure can be calculated. The method allows a very fast analysis so that several analyses may be performed in the time necessary for a single finite element run, so that optimum design may be achieved. Last, but not least, all the model constants are traditional geotechnical parameters, which can be evaluated in the conventional way.

## 2. The spring model

Soil behaviour is nonlinear and irreversible. In an unloading reloading cycle, however, soil can be considered to behave as an elastic material, as a first approximation. Soil stiffness for virgin loading is markedly different from that pertaining to unloading reloading. The level at which virgin loading occurs is not fixed but increases with the maximum stress ever experienced. The state of allowable stresses is limited by the Coulomb Mohr failure condition.

All the information on the fundamental soil behaviour described in the previous paragraph have been condensed into the model. It is assumed that the relation between the force in the i-th spring,  $\textbf{S}_{i}$  ,and the displacement,  $\textbf{s}_{i}$  ,is given by

$$S_i = K_i s_i$$
 (1)

where 
$$K_{i}$$
 is given by
$$K_{i} = a \frac{E_{si}^{i} t_{i}}{L}$$
(2)

Consider first the soil stiffness, E, which is equal to the Young modulus for a linear elastic material. Since soil is not elastic, E can be equal either to E or to E depending on the state of stress and the stress increment direction. Ep is the stiffness for virgin loading. The soil is characterized by such a stiffness only if the state of stress is equal to the maximum past stress and the stress is increasing. Fig.1 better explains this point. Assume that the sheetpile soil interaction is a plane strain problem and that the vertical and horizontal stresses are principal stresses. Thus the relevant part of the state of stress is fully identified by two parameters,  $\sigma_{V}^{\prime}$  and  $\sigma_{H}^{\prime}$ . If  $\sigma_{Vp}^{\prime}$  and  $\sigma_{H}^{\prime}$  are the maximum past vertical and horizontal stresses, it is possible to define in the plane  $\sigma_{V}^{\prime}$ ,  $\sigma_{H}^{\prime}$  a domain of elastic behaviour which is limited by the Coulomb failure lines and the lines  $\sigma_{V}^{\prime} = \sigma_{Vp}^{\prime}$ ,  $\sigma_{H}^{\prime} = \sigma_{Hp}^{\prime}$ . Assume also for the sake of simplicity, that soil is grahular. Stress increments such as AB, CD, Ef are all characterized by the unloading reloading stiffness E , while GI is linked to E . Since for this latter increment  $\sigma_{Vp}^{\prime}$  becomes equal to  $\sigma_{V1}^{\prime}$ , there is an expansion of the elastic domain. Note however that, since  $\sigma_{H}^{\prime}$  does not change,  $\sigma_{Hp}^{\prime}$  remains constant, so that hardening is anisotropic.

The two failure conditions are given by

$$\sigma_{H}^{\prime}=K_{p}\sigma_{V}^{\prime}$$
 (4)

where K<sub>A</sub> and K<sub>p</sub> are the active and passive earth pressure coefficients. In the following, K<sub>A</sub> will be assumed equal to the Rankine value, while the value of K<sub>p</sub> will be chosen in such a way to implicitly take into account friction between soil and wall. Indeed, measured values for  ${\rm K}_{\rm p}$  are quite larger than the Rankine passive coefficient, in all the examined cases.

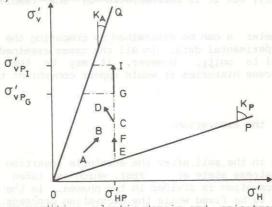


Fig.1 Failure condition, elastic domain and anisotropic hardening for granular soil. The well have becomes at seasts testings and some alignments

Note that E  $_{\rm ur}$  and E  $_{\rm need}$  not to be constant. They can be made to depend on the value of the mean pressure p', for instance, in the following way

$$E_{ur} = R_{ur} \left(\frac{p}{p}\right)^{n}$$

$$E_{ep} = R_{ep} \left(\frac{p}{p}\right)^{n}$$

$$(5)$$

$$E_{p} = R_{p} \left( p / p_{p} \right)^{n} \tag{6}$$

where p is a reference pressure and R , R , n are experimental constants. Also, soil stiffness may be taken variable with stress ratio, as in the hyperbolic stress strain relation, for example. In the following, however, for the sake of simplicity, we shall assume that that soil stiffness does not depend on stress ratio.

Of the other parameters which characterize the spring stiffness, t; simply the influence region of the i-th spring, or the semidistance between spring (i-1)th and spring (i+1)th, a is a non-dimensional constant and L is a geometric quantity which is characteristic of the problem. The latter should take account of the width of the region of soil which is involved in the movement of the sheetpile. It will then be different for the active and the passive zone and will depend on the depth of excavation. It will be assumed that for the springs at the back of the sheetpile

$$L_{\rm M} = \frac{2}{3} H' \tan(45^{\circ} - \Phi/2)$$
 (7)

while in front of it

$$L_{V} = \frac{2}{3} (H' - D) \tan(45^{\circ} - \Phi/2)$$
 (8)

where  $\Phi$  is the effective friction angle of the soil, D is the depth of the excavation, and H' is the effective length of the wall, that is

while H is the total length. The assumption that only the soil up to a depth equal to twice the excavation is involved in the deformation of the wall is arbritary, but it is corroborated by the results of the numerical analysis.

Finally, the parameter a can be determined by comparing the calculated results with the experimental data. In all the cases examined up to now a can be taken equal to unity. However, it may be that from the analysis of other case histories it would appear convenient to choose a different value.

### 3. The modelling of the excavation.

The state of stress in the soil after the sheetpile insertion is assumed to be equal to the stress state at rest, which is taken as known. Each step of the excavation is divided in two phases. In the first one, the wall is assumed to be fixed while the excavation proceeds. There is only a variation of the state of stress in the springs in front of the sheetpile since the vertical stress is reduced and the horizontal stress will vary according to the law

$$\sigma_{H}^{1} = \sigma_{V}^{1} \times \sigma_{O}^{1} = \sigma_{V}^{1} \times \sigma_{O}^{1} = \sigma_{O}^{1} \times \sigma_{O$$

where K is the coefficient of earth pressure at rest for the soil in the virgin condition while m is an empirical coefficient, ranging between 0 and 1, which is generally close to.5. In the following, however, for the sake of simplicity m will be taken equal to zero so that  $\sigma'_{\text{H}}$  =const in this first phase. Note however that the ratio between  $\sigma'_{\text{H}}$  and  $\sigma'_{\text{V}}$  should always be less than  $K_p$ . Near the base of the excavation  $\sigma'_{\text{V}}$  is very small so that Eq.10 will be no more valid and

$$\sigma'_H = K_P \sigma'_V$$
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Let now the wall free to move. Since the state of stress in the springs is no more self-equilibrated the wall will depart from its initial position until a new equilibrium position is achieved. A first guess on the stiffness of the springs is necessary. At the end of each step it is checked whether the calculated displacement is compatible or not with the assumed value of the stiffness. In case it is not, the step is erased, the value of the stiffness is changed and a new step is forwarded. Since the number of springs is generally low, this iteration procedure is fast.

When convergence is achieved a new excavation step can be performed first by fixing the wall and altering the state of stress and then performing the iterations towards a new equilibrium position. When the

excavation is deep enough it is no more possible to reach convergence. This situation is associated with failure.

With such a procedure it is easy to model the anchoring system as well. At the appropriate depth a new linear elastic spring is inserted which has the stiffness and pretension of the anchor. The pretension causes a backward movement of the wall and a rearrangement of the state of stress in the soil springs. When the excavation proceeds, the anchor behaves as an additional (elastic) spring, of appropriate stiffness,, varying its tension at each step of the excavation. In this way, it is then possible to follow the tension variation in the anchoring system.

At each step of the excavation, when the equilibrium position is achieved, it is easy to calculate bending moments and shears within the wall.

The effects of the cohesion, surcharge, overconsolidation, water pressures, can be easily taken into account with simple artifices. Settlements of the soil can be evaluated following a procedure suggested by Bransby and Milligan (1975) once the dilatancy of the soil at failure is known.

4. Comparison with experimental data on reduced scale models.

The first example considered involves a model of a cantilever sheetpile wall. The sand employed is Ticino sand whose mechanical characteristics have been thoroughly described by Jamiolkowski et al (1985). The results of the tests performed are reported in Dolci et al (1986). Fig. 2 shows a comparison between the calculated and observed results of displacements versus depth of the excavation for dense and loose sand. The parameters employed in the model have been derived from triaxial test data.

Fig.3 shows a comparison between calculated and experimental data for a model wall rotated about the toe. Sand is dense rounded Leighton Buzzard and the tests have been performed by James and Bransby (1970). This time, due to lack of information, some of the parameters have been guessed on the basis of the experimental results. It is noteworthy that the model predictions match the soil pressures variation with depth, which is very far from the predictions of Rankine's theory.

Fig.4 shows pressures and bending moments for an anchored sheetpile with different wall stiffness. For comparison, the results predicted by means of Blum's theory are also shown(dotted lines). The trend of the computed results is in clear agreement with the results obtained by Rowe (1955) and the measurements of Rowe and Briggs (1961).

5. CONCLUSIONS. Does to weered in thew stre-double

The comparisons performed are generally successfull. The parameters employed are nothing more than coefficients of earth pressures and stiffness moduli, different for virgin loading and unloading; sheetpiles with any number of anchor levels and struts can be analysed. The numerical procedure employed is very fast so that a design

optimization is possible at a quite reasonable cost. It is the opinion of the authors that this method could be a valid help to the design of anchored retaining structures.

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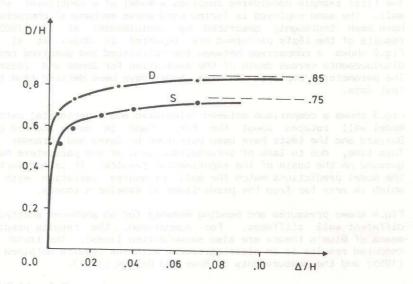


Fig. 2 Relation between depth of excavation and maximum horizontal displacement for a cantilever sheet-pile wall in dense (D) and loose (S) sand: comparison between calculated (solid curves) and experimental (dots) results Experimental data from Dolci et al. (1986).

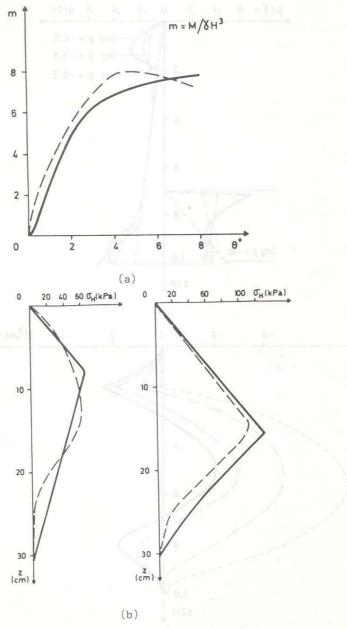


Fig.3 Comparison between experimental (doted lines) and calculated (solid lines) results for a wall rotated about the toe against a mass of dense sand. (a) relation between applied torque and wall rotation,  $\vartheta(b)$  horizontal pressures for  $\vartheta$ = 1° (c) horizontal pressures for  $\vartheta$ = 3°. Experimental data from James and Bransby (1970).

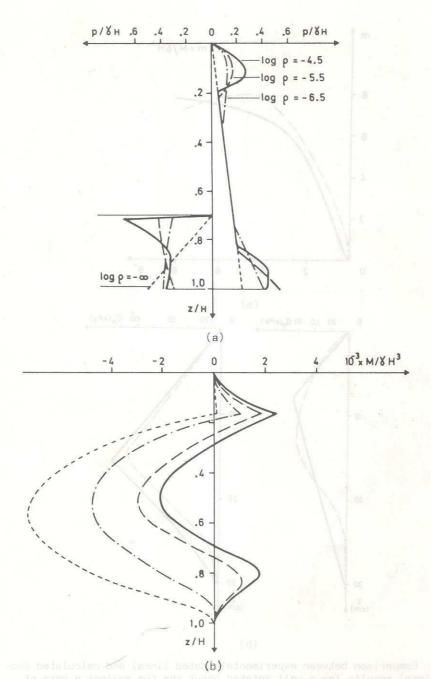


Fig.4 Horizontal pressures (a) and bending moments (b) for a single anchored sheet-pile wall in sand with different flexibility numbers. ( $\rho$  in m/N mm. The curves for log  $\rho$ = - $\infty$  have been obtained by means of the Blum's theory).