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Edited by
G.SWOBODA
Institute of Structural Engineering, University of Innsbruck

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An application of a strainhardening model to the design of tunnels in sand

E.Botti
Metropolitana Milanese, Milan, Italy
G.Canetta
Center of Structural Analysis, Milan, Italy
R.Nova
Milan University of Technology, Italy
R.Peduzzi
Milan, Italy

ABSTRACT: The paper describes the finite element analysis of an urban railway tunnel in a sandy soil improved by grouting. The constitutive model employed to describe the behaviour of natural sand is elastic plastic strainhardening. To allow an easy determination of constitutive parameters from conventional in situ tests, the model has been conceived to be characterized by four parameters, two elastic and two plastic. However, the choice of the yield function is such that the dependence of the behaviour on the Lode angle is correctly matched. The model for grouted sand is elastic perfectly plastic. The grouting pressures induced in the soil are modelled as a field of selfequilibrated stresses. The results obtained are compared with the results of previous analyses performed with elastic perfectly plastic models. It is shown that the analysis with the strainhardening model gives closer prediction of the measured tunnel behaviour.

1. INTRODUCTION

In the last decade a considerable number of constitutive models of soil behaviour have been formulated. Many of them are able to reproduce the experimental results with reasonable accuracy, not only qualitatively but also quantitatively. A long way has been run since the appearance of the original Cam-Clay model (Schofield and Wroth (1968)).

Despite the evident success in describing the observed soil behaviour, however, these models are seldom used for the analysis of engineering problems. The reason is that soil behaviour is overwhelmingly complex and, accordingly, models able to reproduce all its aspects enjoy a complicated theoretical structure and are characterized by quite a number of constitutive parameters. Often, only large computers may cope with theoretical complexity and it is virtually impossible to determine the appropriate values of the constants from routine tests, especially when a non-cohesive soil is involved in the problem to be analysed.

Because of such difficulties, practicing engineers tend to ignore the most advanced theories and make use of simple elastic-perfectly plastic laws which are characterized by few parameters and may be determined by means of simple in-situ tests. In this way, all the major advances in the understanding of soil behaviour of the last twenty years have little or no feedback on geotechnical engineering.

For practical purposes, however, the most advanced models are possibly unduly complicated. If loading is quasistatic and there are no unloading-reloading cycles, even much simpler elastic-plastic strainhardening models give reasonable predictions. Since the theoretical structure of such models is simple and the number of consitutive parameters is limited, they may be successfully used in practice even if large computers are not available and the information on soil properties is scanty and comes form SPT and CPT tests only.

The aim of this paper is to present a simplified model for a granular, non-cohesive material, and to show the results of a practical application. The model is elastic-plastic strainhardening and is characterized by four consitutive parameters which may be related to SPT blows or other results of in-situ tests by means of empirical relations. In fact the parameters are linked to traditional geotechnical constants such as friction angle or elastic moduli for which such relations already exist.

The application concerns the excavation of an urban tunnel in sand. Soil properties have been improved by grouting. The results obtained are compared with those derived by means of an elastic-perfectly plastic analysis and both are compared with experimental data whenever possible.

It is shown that the strainhardening model reproduces better than the elastic-plastic one the observed displacement pattern.

2. THE STRAINHARDENING MODEL 'LAMBER'

The constitutive model that will be employed for the sake of convenience will be referred to as 'Lamber', from the name of a small river which touches Milan. The model is elastic plastic strainhardening (or softening) and is conceptually similar to Cam Clay. The main difference lies in the choice of the expression for the yield function that, at variance with Cam Clay, is function of the three stress invariants. The yield function expands or contracts depending on the value of the hardening modulus which may be either positive or negative. Admissible stress states are bounded by a limiting surface which coincides with the Matsuoka-Nakai (1974) failure condition. This latter criterion may be written in the following way:

$$3/2(\xi-1)J_{2\eta} - \xi J_{3\eta} - 3(\xi-3) = 0 \qquad (1)$$

where ξ is a constitutive parameter, while $J_{2\eta}$ and $J_{3\eta}$ are the second and third invariants of the tensor η_{ij} defined as

$$\eta_{ij} = s_{ij}/p' \qquad (2)$$

The tensor s_{ij} is the stress deviator while p' is the hydrostatic effective stress. The second and third invariants are defined as:

$$J_{2\eta} = \eta_{ij}\eta_{ij} \qquad (3)$$

$$J_{3\eta} = \eta_{ij}\eta_{jk}\eta_{ki} \qquad (4)$$

The yield function is given by:

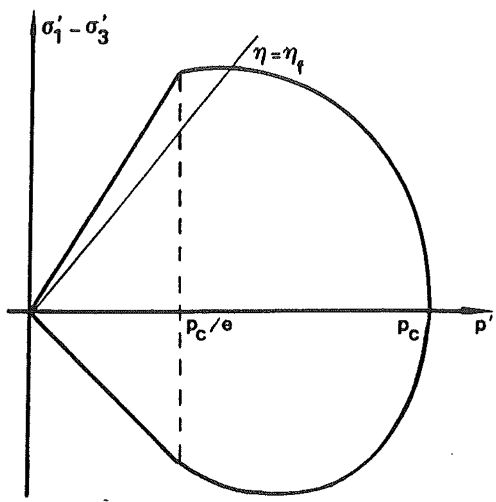
$$f = 3/2(\xi-1)J_{2\eta} - \xi J_{3\eta} + 3(\xi-3)\ln\frac{p'}{p_c} \qquad (5)$$

where p_c is the maximum hydrostatic preconsolidation pressure, which depends on the history of the soil element considered. The yield function is defined only for values of p' larger than p_c/e. When $p'=p_c/e$ Eq. (1) and Eq. (5) coincide. Stress states for which p' is less than p_c/e cannot lie on the yield surface. The complete picture of the yield surface and of the limiting surface is given in fig. 1.

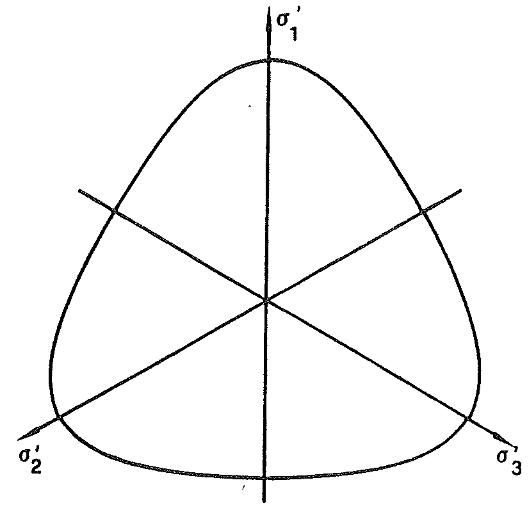
It is assumed that p_c depends only on the plastic volumetric strain experienced:

$$p_c = p_{c0} \exp(v^p/\chi) \qquad (6)$$

p_{c0} is a dummy reference pressure while it is easy to recognize that χ is a plastic logarithmic volumetric compliance. In a purely hydrostatic test χ would be the slope of



a) 'triaxial' plane



b) deviatoric plane $\sigma_2=\sigma_3$

Fig. 1. Yield and limiting surface.

the plot plastic volumetric strain, natur logarithm of hydrostatic pressure, fig. 2

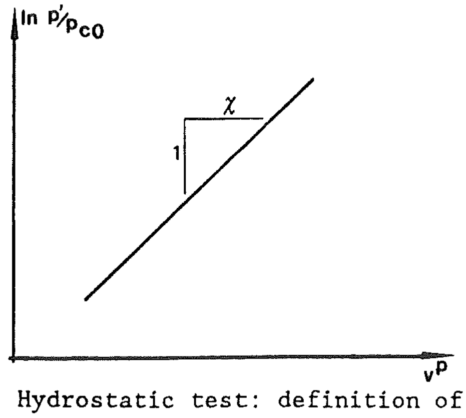


Fig. 2. Hydrostatic test: definition of χ.

The plastic potential is assumed to coi cide with the yield function. Although th has been demonstrated to be far from expe imental reality, the assumption of the validity of the normality rule is taken f limiting the number of the constitutive parameters and to ensure the symmetry of the stiffness matrix of the soil element, what allows well known computational advantages.

Plastic strain rates can be then obtained as

$$\dot{\epsilon}^p_{ij} = \frac{1}{H}\frac{\partial f}{\partial \sigma'_{ij}}\frac{\partial f}{\partial \sigma'_{hk}}\dot{\sigma}'_{hk} \qquad (7)$$

where the hardening modulus H is derived via the Prager's consistency rule:

$$H = \frac{9(\xi-3)}{\chi p'}\left(\xi J_{3\eta} - (\xi-1)J_{2\eta} + \xi-3\right) \qquad (8)$$

For a virgin soil, i.e. normally consolidated, failure will occur when H=0. In axisymmetric conditions ($\sigma_2=\sigma_3$) the stress ratio η, defined as

$$\eta = (\sigma'_1-\sigma'_3)/p' \qquad (9)$$

takes the value η_f that is linked to ξ via Eq. (8). It is easy to show that

$$\xi = \frac{3 - 2/3\eta_f^2}{2/9\eta_f^3 - 2/3\,\eta_f^2 + 1} \qquad (10)$$

Since η_f is linked to the apparent friction angle ϕ' by the relation

$$\eta_f = \frac{6\sin\phi'}{3-\sin\phi'} \qquad (11)$$

also the constitutive parameter ξ may be linked to ϕ' by a one to one correspondence.

Elastic strains are linked to effective stresses by means of a nonlinear elastic law. It is assumed that the bulk modulus K linearly varies with p', while the shear modulus G is taken as constant.

Four parameters and the dummy constant p_{c0} fully characterize the model. This latter constant has little practical relevance since the initial state of stress due to selfweight is normally associated with zero strains. In each point p_{c0} may be simply taken as the initial maximum hydrostatic stress ever experienced. The parameter ξ may be easily determined from the assumed value of the friction angle. This in turn may be determined by means of empirical relationships with the SPT blow count or the Dutch cone penetration resistance.

The shear modulus may be chosen as for a purely elastic analysis, while the determination of the bulk modulus and of the plastic compliance is more difficult. In fact several empirical relations exist that allow to determine what is normally taken as an 'elastic modulus'. Since the actual soil behaviour is not elastic, however, the 'elastic modulus' determined by means of the empirical relations is in fact a sort of weighted average between the proper elastic stiffness, which is associated with conditions of unloading / reloading, and the elastic plastic stiffness associated to virgin loading. Moreover, since the elastic plastic stiffness is stress-path dependent, it is clear that an unambiguous recipe for determinig χ and K from SPT or CPT tests does not yet exist. In the following, therefore, the parameters χ and K will be chosen on the basis of previous experience matured in modelling soil behaviour in fully controlled laboratory test. From results in isotropic loading tests, elastic, i.e. reversible, volumetric strains may vary between 1/3 and 2/3 of total volumetric strains.

On the other hand χ may be linked to friction angle by another empirical relation such as:

$$\chi = (1.9-\eta_f)/62 \qquad \eta_f<1.9 \ (\phi'<46.2°) \qquad (12)$$

Eq. (12) has been derived on the basis of few experimental data shown in fig. 3 and should be used cautiously. It should be

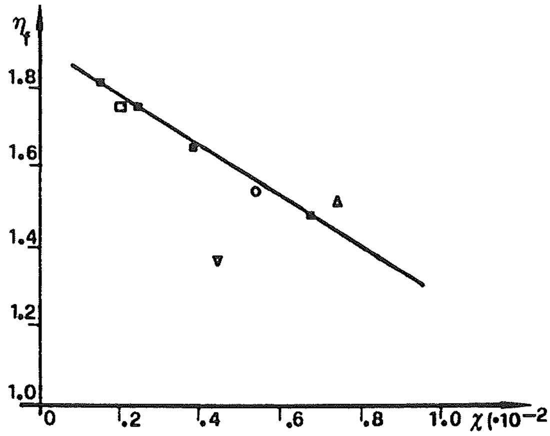


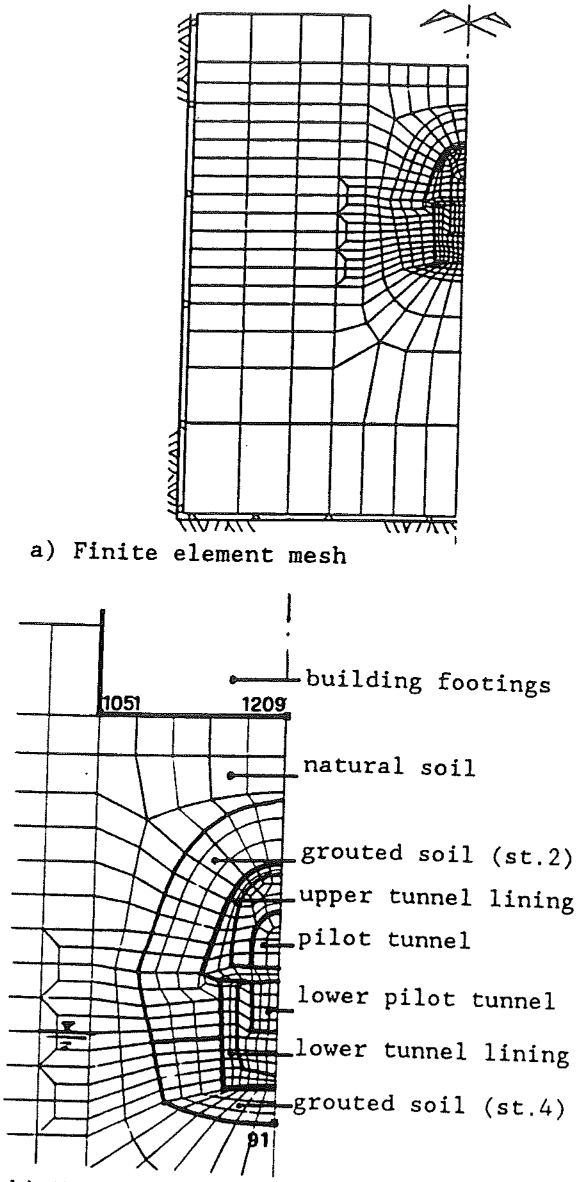
Fig. 3. Empirical relation between χ and η_f.

then considered as a very rough first approximation. More adequate relations will be proposed when enough experience will be gained on actual behaviour of real structures.

3. THE MODELLING OF A TUNNEL EXCAVATION IN SAND

The constitutive law presented has been employed for the finite element analysis of the excavation of an urban tunnel of the new line of the Milan Underground. To fit the narrow streets of the historic part of the town, a special profile with superposed rails has been chosen. A typical section of the tunnel is shown in fig. 4, where the 550 isoparametric 8-noded element mesh is also plotted.

The soil at site is alluvional sand and gravel. A typical stratigraphic profile together with the measured SPT blow counts is shown in fig. 5.



a) Finite element mesh

b) Mesh details

Fig. 4. Section of tunnel studied and finite element mesh.

The tunnel excavation has been carried out in the following steps:

1. Excavation of a 3.3 m wide pilot tunnel with shotcrete lining.
2. Grouting of a 3.5 m thick layer of soil surrounding the excavation profile of the upper tunnel and the shoulders of the lower one; grouting is not extended below the invert; injections are performed from the pilot tunnel; cement grouting is complemented with silicate additives where necessary (in sand and sandy gravel).
3. Excavation of the upper tunnel, with provisional steel ribs and shotcrete lining;
4. Grouting of a 2 m thick layer of soil below the profile of the invert of the lower tunnel; injections are performed from the floor of the upper tunnel; after the completion of the step, grouted soil forms a continuous ring surrounding the excavation profile.
5. Casting of the upper tunnel floor slab and crown arch lining.
6. Excavation of a 3 m wide pilot tunnel below the upper part floor slab; the bottom of this tunnel coincides with water level; walls have no lining and their stability relies upon apparent cohesion.
7. Lowering and shoulder excavation of the lower tunnel; walls are provisionally lined with steel struts and shotcrete.
8. Casting of the lining of invert arch and walls of the lower tunnel.

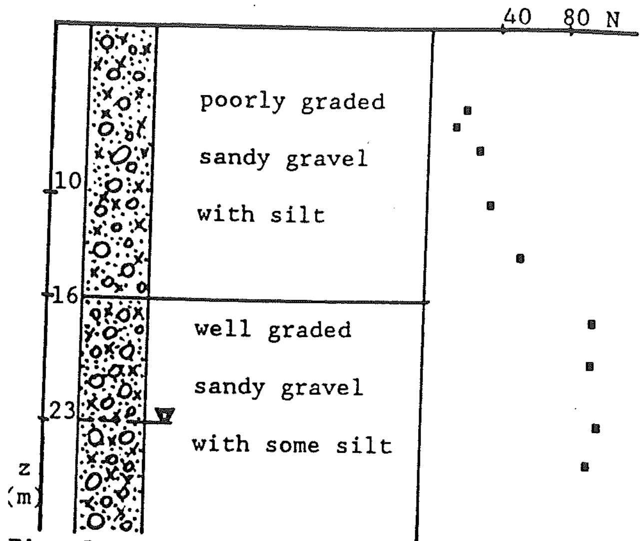


Fig. 5. Typical simplified soil profile.

The excavation procedure has been simulated by means of 16 loading steps on a plane strain model. Starting from imposed geostatic conditions, excavation, grouting and concrete casting are modelled with 'death' and 'birth' of finite elements and removal and application of respective weights.

Grouting improves mechanical performances of soil and introduces a self equilibrated force system as a consequence of injection pressures. This is simulated by means of enhanced material parameters and introducing an anelastic isotropic strain to prestress grouted soil arches. The numerical value of the imposed strain is determined in a semi-empirical way to obtain node displacements in agreement with measured surface heave.

Because of the plane strain approach adopted, in the crown excavation phase the crown provisional lining is supposed to begin acting simultaneously with crown excavation; in this way the settlement is neglected which takes place ahead from excavation front, as a consequence of front wedge decompression. To evaluate this settlement a single step analysis is performed in which crown excavation takes place with no lining; a fraction of the node displacements calculated in this way is then added to the node settlements of the crown excavation phase as calculated in the main analysis. According to experimental observations in similar conditions (e.g. Peduzzi et al. (1986)) this fraction has been taken equal to one third.

To see the influence of the constitutive model adopted on the results, various analyses with two different constitutive models have been conducted. The former is the one described in this paper, while the latter is a conventional elastic- perfectly plastic model with associate flow rule and Drucker-Prager yield condition.

Indeed several similar tunnels of the Milan Underground had previously been studied with the latter model. The calculated results were in substantial agreement with the measured settlements; however, during the excavation steps, the calculated heave was far larger than the observed, as a consequence of the low elastic modulus adopted to represent both virgin compression and unloading / reloading behaviour. The new material model has been conceived in order to avoid this discrepancy.

Different procedures have been followed to determine the constitutive parameters from available information on soil properties.

The elastic-perfectly plastic model is in this case governed by three parameters, the Young's modulus E, the Poisson's ratio ν and the effective friction angle ϕ'. The Young's modulus has been derived by means of the following empirical relationship (D'Appolonia et al. (1970)):

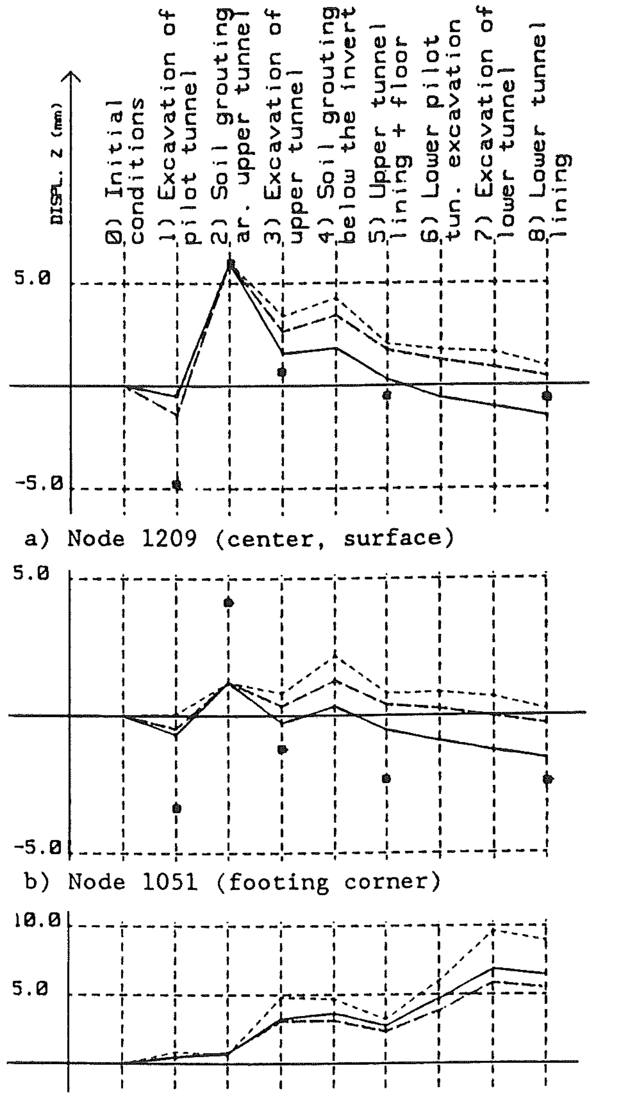
$$E = 21.6 + 1.06N \tag{13}$$

where N is the SPT blow count and E is in MPa. The Poisson's ratio has been taken equal to .25 which is a common value for the alluvional soil considered, while the friction angle has been derived from the empirical relation between ϕ' and N proposed by Bazaraa (1967), that takes the influence of overburden pressure into account. As far as strength is concerned, the soil can be conveniently subdivided in an upper layer 16 m thick with $\phi'=35°$ and a lower layer with $\phi'=38.5°$.

The parameters for the Lamber model were calculated as follows. From the values of the friction angles calculated as above, the values of η_f have been determined from Eq. (11). Then the values of ξ and χ have been derived form Eq.s (10) and (12) respectively. The elastic bulk modulus has been taken in such a way to be three times the current elastic-plastic bulk modulus. Finally the elastic shear modulus was chosen in such a way that the initial apparent Poisson's ratio be .25.

For all the analyses performed the grouted soil in supposed to be elastic-perfectly plastic. The elastic modulus E is taken to be 210MPa. Although experimental results show much higher values, this value was taken as an average to account for macroscopic discontinuity in soil injection. The other parameters were taken as $\nu=0.25$, $\phi'=35°$ while the cohesion is .2 MPa (average value as for E).

Fig. 6a,6b and 6c show the calculated vertical displacement of three critical points versus in-situ measured settlements in the eight construction phases; fig.s 7 and 8 represent typical stress and displacement conditions.



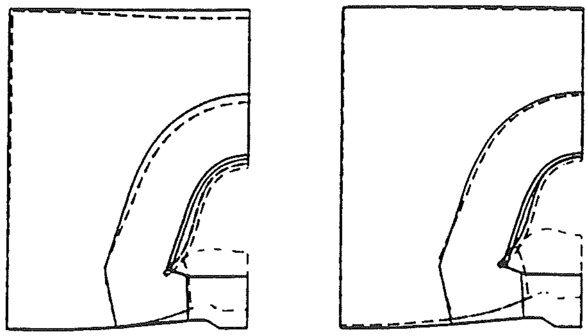
a) Node 1209 (center, surface)

b) Node 1051 (footing corner)

c) Node 91 (center, below invert)

Fig. 6. Comparison of dispacements.

As it can be seen from fig.s 6a,6b, which relate to surface points, the settlement predictions obtained with Lamber (solid line) are closer to experimental results (single square symbols) than those of the elastic-perfectly plastic model (short dash line); fig. 6c, in which the heave of a point below the excavation profile is plotted, shows the unrealistic higher swelling of the elastic-perfectly plastic model with respect to that predicted with Lamber. This behaviour is emphasized in fig. 7, which represents the displaced patterns in the crown excavation phase for the two models (amplif. 200 times).



a) Lamber model b) el.-perf.pl. model

Fig. 7. Deformed shape comparison (step 3)

The predictions of the elastic-perfectly plastic model could be improved with a different choice of material parameters. A third analysis has therefore been run, in which the elastic modulus of natural soil has been chosen in such a way to be equivalent to the bulk modulus used in the analysis with Lamber; also the friction angle has been reduced so that, in plane strain conditions, the limiting state is close to that of Lamber. The results are plotted with long dash lines on the same fig.s 6a,6b and 6c. The heave of the point at depth is now very close to Lamber's (the deep layer's behaviour being elastic). The surface settlements do not depart much from those calculated with the previous elastic-perfectly plastic analysis and are thus far from the measured ones. At least, in the problem considered, the strainhardening model proves then to give better predictions.

4. CONCLUSIONS

In this paper, a simple strainhardening constitutive model has been presented, which has been conceived to allow the numerical analysis of engineering problems. In order to do that, the complexity of the mathematical structure of the model and the number of constitutive parameters has been kept to the minimum. In the application presented it is shown that the constitutive parameters can be derived by combining the scanty information coming from SPT test, some empirical relationships between the blow count and the other geotechnical parameters and previous experience gained

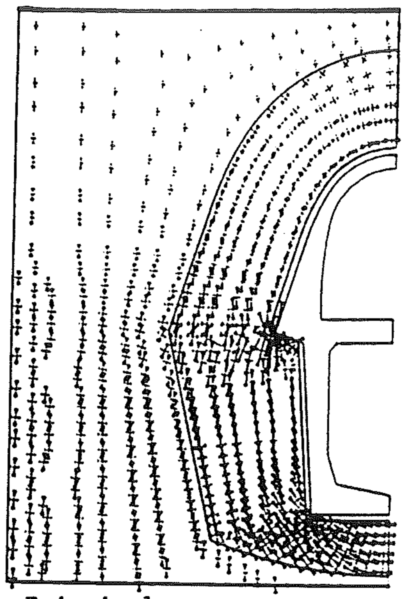


Fig. 8. Principal stresses.

in modelling laboratory soil behaviour.

The results obtained in the analysis of a tunnel with superposed rails are compared with the results obtained by means of a more classical analysis utilizing an elastic-perfectly plastic constitutive model. The former analysis gives predictions which are closer to observed data. The model presented constitutes then a promising tool for the solution of boundary value problems of practical interest.

5. ACKNOWLEDGEMENTS

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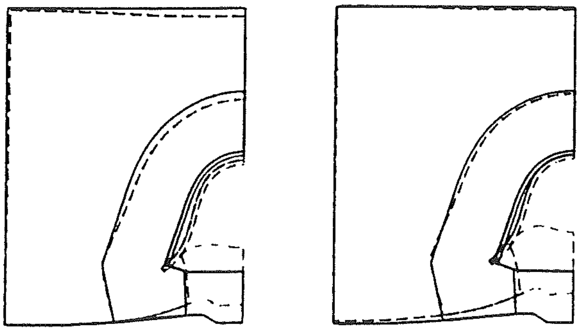

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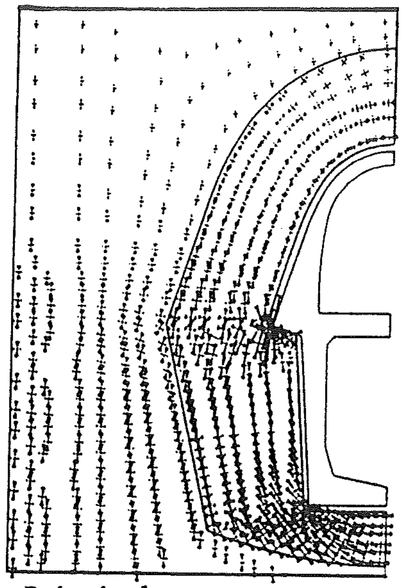

Fig. 8. Principal stresses.

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